

# Holographic Superconductors in Gauss-Bonnet gravity with Born-Infeld electrodynamics

Jiliang Jing\*, Liancheng Wang, Qiyuan Pan, and Songbai Chen

*Institute of Physics and Department of Physics,*

*Hunan Normal University, Changsha,*

*Hunan 410081, P. R. China*

*and*

*Key Laboratory of Low Dimensional Quantum Structures*

*and Quantum Control of Ministry of Education,*

*Hunan Normal University, Changsha,*

*Hunan 410081, P. R. China*

## Abstract

We investigate the holographic superconductors in Gauss-Bonnet gravity with Born-Infeld electrodynamics. We find that the Gauss-Bonnet constant, the model parameters and the Born-Infeld coupling parameter will affect the formation of the scalar hair, the transition point of the phase transition from the second order to the first order, and the relation connecting the gap frequency in conductivity with the critical temperature. The combination of the Gauss-Bonnet gravity and the Born-Infeld electrodynamics provides richer physics in the phase transition and the condensation of the scalar hair.

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\* Electronic address: jljing@hunnu.edu.cn

## I. INTRODUCTION

The AdS/CFT correspondence [1–3] relates a weak coupling gravity theory in an anti-de Sitter space to a strong coupling conformal field theory in one less dimensions. Recently it has been applied to condensed matter physics and in particular to superconductivity [4, 5]. In the pioneering papers Gubser [4, 5] suggested that near the horizon of a charged black hole there is in operation a geometrical mechanism parameterized by a charged scalar field of breaking a local  $U(1)$  gauge symmetry. Then, the gravitational dual of the transition from normal to superconducting states in the boundary theory was constructed. This dual consists of a system with a black hole and a charged scalar field, in which the black hole admits scalar hair at temperature lower than a critical temperature, but does not possess scalar hair at higher temperatures [6]. In this system a scalar condensate can take place through the coupling of the scalar field with the Maxwell field. Much attention has been focused on the application of AdS/CFT correspondence to condensed matter physics since then [7–18].

Recently, it is of great interest to generalize the investigation to the Einstein-Gauss-Bonnet gravity, which is motivated by the application of the Mermin-Wagner theorem to the holographic superconductors. It was found [19–23] that the higher curvature corrections in general make the condensation of the scalar field harder to form and give larger corrections to the so-called Horowitz’s relation  $\omega_g/T_c \approx 8$  for the conductivity. And then the general holographic superconductor models in Einstein-Gauss-Bonnet gravity are constructed and it is observed that different values of Gauss-Bonnet correction term and model parameters can determine the order of phase transitions and critical exponents of second-order phase transitions [24]. Very recently, the holographic p-wave superconductor models in the Gauss-Bonnet gravity was introduced in the probe limit and the Gauss-Bonnet correction term can also effect the condensation of the vector field [25].

On the other hand, non-linear electrodynamics has been a subject of research for many years. Heisenberg and Euler [26] noted that quantum electrodynamics predicts that the electromagnetic field behaves non-linearly through the presence of virtual charged particles. Born and Infeld [27] presented a new classical non-linear theory of electromagnetism which contains many symmetries common to the Maxwell theory despite its non-linearity. It was found that the Born-Infeld electrodynamics is the only possible non-linear version of electrodynamics that is invariant under electromagnetic duality transformations [28]. Thus, the interest of study for Born-Infeld electrodynamics has been arisen in [29–31]. The static spherically symmetric black holes for the Born-Infeld electrodynamics coupled

to Einstein gravity was derived in Refs. [29, 30]. Within the framework of AdS/CFT correspondence, we studied the effects of the Born-Infeld electrodynamics on the holographic superconductors in the background of a Schwarzschild-AdS black hole spacetime [32].

Motivated by the recent studies mentioned above and the fact that, within the framework of AdS/CFT correspondence, higher-derivative corrections to either gravitational or electromagnetic action in AdS space are expected to modify the dynamics of the strongly coupled dual theory, in this paper we will investigate the behavior of the holographic superconductors in the Gauss-Bonnet gravity with the Born-Infeld electrodynamics in a five dimensional planar black-hole background, and to see how the combination of the Gauss-Bonnet gravity and the Born-Infeld electrodynamics affect the formation of the scalar hair, the phase transition and Horowitz's relation.

The paper is organized as follows. In Sec. II, we explore the scalar condensation in the background of the Gauss-Bonnet black hole by introducing a complex charged scalar field coupling with an electric field obeyed to Born-Infeld electrodynamics. In Sec. III, we study the electrical conductivity and find the ratio of the gap frequency in conductivity to the critical temperature. We summarize and discuss our conclusions in the last section.

## II. SCALAR CONDENSATION

The Einstein-Gauss-Bonnet theory is the most general Lovelock theory in five and six dimensions and the action is described by

$$I_{grav} = \frac{1}{16\pi G} \int_{\mathcal{M}} d^d x \sqrt{-g} \left[ R - 2\Lambda + \hat{\alpha} \left( R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} \right) \right], \quad (2.1)$$

where  $\Lambda = -(d-1)(d-2)/(2L^2)$  is the cosmological constant,  $G$  is the gravitational constant, and  $\hat{\alpha}$  is the Gauss-Bonnet coupling constant. The static spacetime of a neutral black hole in  $d$  dimensional Einstein-Gauss-Bonnet gravity is [33–35]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dx_i dx^i, \quad (2.2)$$

with

$$f(r) = \frac{r^2}{2\alpha} \left[ 1 - \sqrt{1 - \frac{4\alpha}{L^2} \left( 1 - \frac{ML^2}{r^{d-1}} \right)} \right], \quad (2.3)$$

where  $\alpha = \hat{\alpha}(d-3)(d-4)$  and the constant  $M$  is relate to the black hole horizon by  $r_+ = (ML^2)^{1/(d-1)}$ . In the asymptotic region ( $r \rightarrow \infty$ ), we have  $f(r) \sim \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 - 4\alpha/L^2} \right)$ . Thus, we can define the

effective asymptotic AdS scale by  $L_{\text{eff}}^2 = (2\alpha)/(1 - \sqrt{1 - 4\alpha/L^2})$ . The Hawking temperature of the black hole is

$$T = \frac{(d-1)r_+}{4\pi L^2}. \quad (2.4)$$

We now consider the Born-Infeld electrodynamics and the charged scalar field coupled via a generalized Lagrangian

$$S = \int d^d x \sqrt{-g} \left[ \frac{1}{b^2} \left( 1 - \sqrt{1 + \frac{b^2 F^2}{2}} \right) - \frac{1}{2} \partial_\mu \tilde{\psi} \partial^\mu \tilde{\psi} - \frac{1}{2} m^2 \tilde{\psi}^2 - \frac{1}{2} |\mathfrak{F}(\tilde{\psi})| (\partial_\mu p - A_\mu) (\partial^\mu p - A^\mu) \right], \quad (2.5)$$

where  $\mathfrak{F}$  is taken as  $\mathfrak{F}(\tilde{\psi}) = \tilde{\psi}^2 + c_\gamma \tilde{\psi}^\gamma + c_4 \tilde{\psi}^4$  with the model parameters  $c_\gamma$ ,  $\gamma$  and  $c_4$  in order to introduce a general class of gravity duals to superconducting theories that exhibit both first and second-order phase transitions at finite temperature in strongly interacting systems [36, 37], and it reduces to the model considered in [32] if  $c_\gamma$  and  $c_4$  are zero. It should be noted that  $b$  is the Born-Bonnet coupling parameter and the Born-Infeld electrodynamics will reduce to the Maxwell case in the weak-coupling limit  $b \rightarrow 0$ . We can use the gauge freedom to fix  $p = 0$  and take  $\psi \equiv \tilde{\psi}$ ,  $A_t = \phi$  where  $\psi$ ,  $\phi$  are both real functions of  $r$  only. Then the equations of motion are given by

$$\psi'' + \left( \frac{f'}{f} + \frac{d-2}{r} \right) \psi' + \frac{\phi^2}{f^2} \left( \psi + \frac{\gamma}{2} c_\gamma \psi^\gamma + 2c_4 \psi^3 \right) - \frac{m^2}{f} \psi = 0, \quad (2.6)$$

$$\left( \phi'' + \frac{d-2}{r} \phi' \right) (1 - b^2 \phi'^2) + b^2 \phi'^2 \phi'' - (1 - b^2 \phi'^2)^{\frac{3}{2}} (\psi^2 + c_\gamma \psi^\gamma + c_4 \psi^4) \frac{\phi}{f} = 0, \quad (2.7)$$

where a prime denotes the derivative with respect to  $r$ . At the event horizon  $r = r_+$ , we must have

$$\psi(r_+) = -\frac{(d-1)}{m^2 L^2} \psi'(r_+), \quad \phi(r_+) = 0, \quad (2.8)$$

and at the asymptotic AdS region ( $r \rightarrow \infty$ ), the solutions behave like

$$\psi = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}, \quad \phi = \mu - \frac{\rho}{r^{d-3}}, \quad (2.9)$$

with

$$\lambda_\pm = \frac{1}{2} \left[ (d-1) \pm \sqrt{(d-1)^2 + 4m^2 L_{\text{eff}}^2} \right], \quad (2.10)$$

where  $\mu$  and  $\rho$  are interpreted as the chemical potential and charge density in the dual field theory respectively. We take  $\psi_- = 0$  because we can impose boundary condition that either  $\psi_+$  or  $\psi_-$  vanishes [6, 7], and we will focus on  $d = 5$  and  $m^2 L^2 = -3$  here. Thus, the scalar condensate is now described

by the operator  $\langle O_+ \rangle = \psi_+$ . In what following we will present a detail analysis of the condensation of the operator  $\langle O_+ \rangle$  by taking numerical integration of the equations (2.6) and (2.7) from the horizon out to the infinity with the boundary conditions mentioned above.

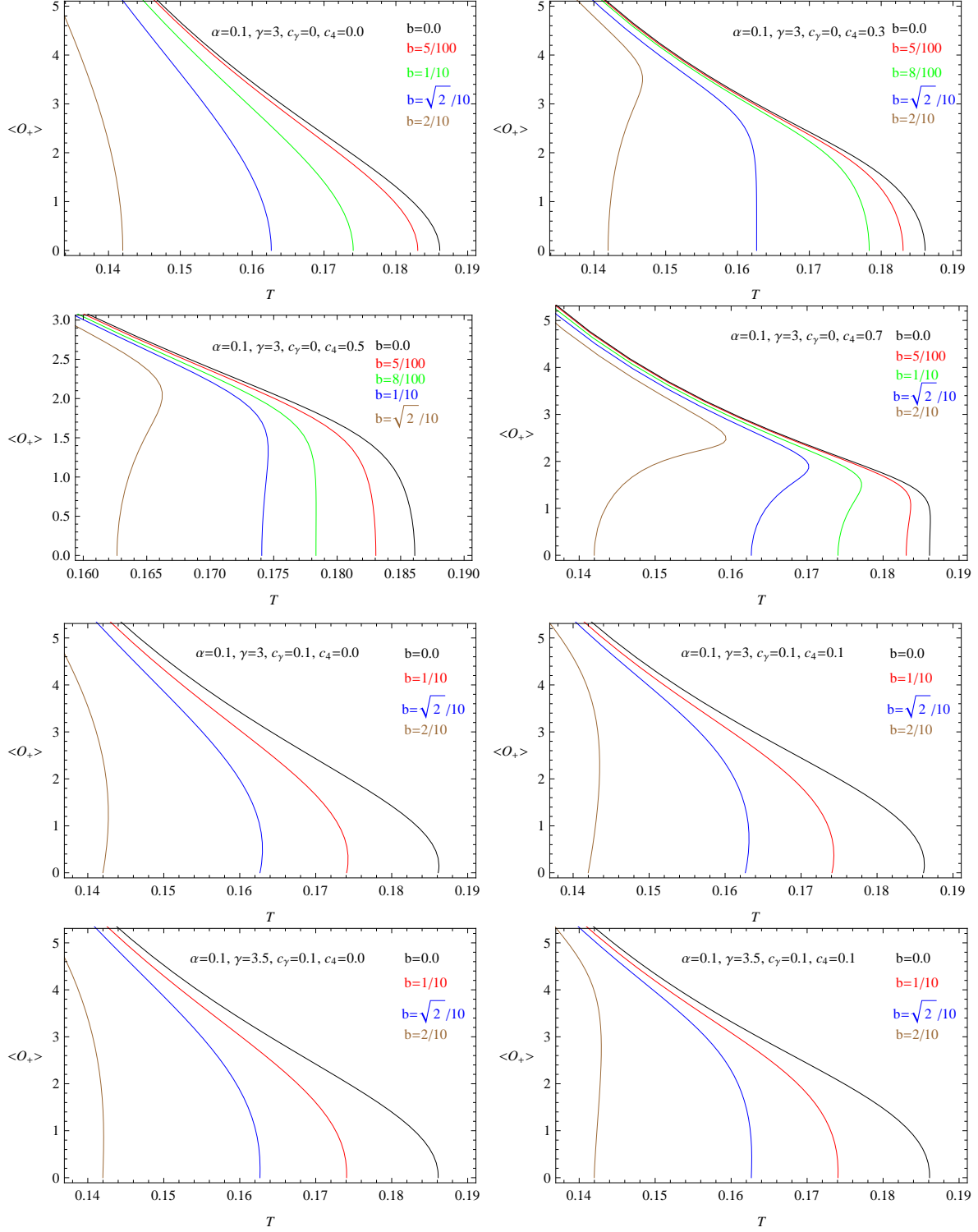


FIG. 1: (Color online) The condensate  $\langle O_+ \rangle$  as a function of temperature with fixed value  $\alpha = 0.1$  for different values of the model parameters ( $c_\gamma$ ,  $\gamma$ ,  $c_4$ ) and Born-Infeld coupling parameter  $b$ , which shows that a different values of these parameters not only change the formation of the scalar hair, but also separate the first- and second-order phase transition.

TABLE I: The critical values of  $T_c$  and  $b_c$  for different  $\alpha$  and  $c_4$ , which can separate the first- and second-order phase transitions for the simple model  $\mathfrak{F}(\psi) = \psi^2 + c_4\psi^4$ . The word “No” in the table corresponds to the inexistence of the critical point.

	$c_4 = 0$	$c_4 = 0.3$	$c_4 = 0.5$	$c_4 = 0.7$	$c_4 = 1.0$
$\alpha = 0.1$	No	$b_c = \frac{\sqrt{2}}{10} \quad T_c = 0.16253$	$b_c = \frac{1}{10} \quad T_c = 0.17456$	$b_c = 0 \quad T_c = 0.18622$	No
$\alpha = 0.0$	No	$b_c = \frac{2}{10} \quad T_c = 0.16566$	$b_c = \frac{\sqrt{2}}{10} \quad T_c = 0.18118$	$b_c = \frac{1}{10} \quad T_c = 0.18973$	No
$\alpha = -0.1$	No	$b_c = \frac{\sqrt{7}}{10} \quad T_c = 0.16824$	$b_c = \frac{2}{10} \quad T_c = 0.18454$	$b_c = \frac{\sqrt{2}}{10} \quad T_c = 0.19636$	$b_c = 0 \quad T_c = 0.20636$

We present in Fig. 1 the influence of the parameters  $(c_\gamma, \gamma, c_4)$  and  $b$  on the condensation with fixed values  $m^2 L^2 = -3$  and  $\alpha = 0.1$ . In fact, the different choices of  $\alpha$  can not qualitatively change our results. We know from the figure that the Born-Infeld coupling parameter and the model parameters have obvious different effects on the critical temperature. If we fix the model parameters  $(c_\gamma, \gamma, c_4)$ , we note that the critical temperature becomes smaller as the Born-Infeld coupling parameter  $b$  increases for two types of phase transitions, i.e., the scalar hair can be formed harder for the larger  $b$ . However, the story is completely different if we fix the Born-Infeld coupling parameter  $b$ . For the cases of second phase transition, the critical temperature keeps as a constant with the increase of  $(c_\gamma, \gamma, c_4)$ . That is to say, the formation of the scalar hair does not affect by model parameter  $(c_\gamma, \gamma, c_4)$ . But for the cases of first phase transition, the critical temperature is larger with the increase of  $(c_\gamma, c_4)$  or decrease of  $\gamma$ , which means that the scalar hair can be formed easier for the larger model parameter  $(c_\gamma, c_4)$  or smaller  $\gamma$ . It should be pointed out that we define the critical temperature  $T_c$  just as Franco *et al* for the first phase transition [37].

From Fig. 1 we also find that, for a simple model  $\mathfrak{F}(\psi) = \psi^2 + c_4\psi^4$  with fixed  $c_4$  ( $c_4 > 0$ ), there is a phase transition from the second order to the first one as we increase value of  $b$ . In table I, we list the critical values of  $b_c$  and  $T_c$  which separate the second order and the first order phase transitions for selected  $\alpha$  and  $c_4$ . Note that the word “No” in this table corresponds to the inexistence of the critical values of  $b_c$  and  $T_c$ , i.e., the phase transition is always of the second order if  $c_4 = 0$  with different  $\alpha$  but the first order if  $c_4 = 1.0$  with  $\alpha = 0.0$  and  $0.1$ . We learn from the figure and the table that both  $b_c$  and  $T_c$  decrease as  $\alpha$  increases for fixed  $c_4$ , and  $b_c$  decreases but  $T_c$  increases as  $c_4$  increases for fixed  $\alpha$ . Thus, the Gauss-Bonnet constant  $\alpha$ , the model parameter  $c_4$  and the Born-Infeld coupling parameter  $b$  provide richer physics in the phase transition.

### III. ELECTRICAL CONDUCTIVITY

In the study of (2+1) and (3+1)-dimensional superconductors, Horowitz *et al.* [8] got a universal relation connecting the gap frequency in conductivity with the critical temperature, which is described by

$$\frac{\omega_g}{T_c} \approx 8, \quad (3.1)$$

with deviations of less than 8%. This is roughly twice the BCS value 3.5 indicating that the holographic superconductors are strongly coupled. However, the authors in Refs. [19, 20] found that this relation is not stable in the presence of the Gauss-Bonnet correction terms. We now examine this relation for the Gauss-Bonnet gravity with the Born-Infeld electrodynamics.

In order to compute the electrical conductivity, we should study the Born-Infeld electromagnetic perturbation in this Gauss-Bonnet black hole background, and then calculate the linear response to the perturbation. In the probe approximation, the effect of the perturbation of metric can be ignored. Assuming that the perturbation of the vector potential is translational symmetric and has a time dependence as  $\delta A_x = A_x(r)e^{-i\omega t}$ , we find that the motion equation for the Born-Infeld electrodynamics in the Gauss-Bonnet black hole background reads

$$\begin{aligned} & \left( A_x'' + \frac{f'}{f} A_x' + \frac{d-4}{r} A_x' + \frac{\omega^2}{f^2} A_x \right) (1 - b^2 \phi'^2) \\ & + b^2 \phi' \phi'' A_x' - \left( 1 - b^2 \phi'^2 \right)^{\frac{3}{2}} (\psi^2 + c_\gamma \psi^\gamma + c_4 \psi^4) \frac{A_x}{f} = 0. \end{aligned} \quad (3.2)$$

From Eq. (3.2), an ingoing wave boundary condition near the horizon is given by

$$A_x(r) \sim f(r)^{-\frac{i\omega L^2}{(d-1)r_+}}, \quad (3.3)$$

and in the asymptotic AdS region ( $r \rightarrow \infty$ ), the general behavior for  $d = 5$  should be [23]

$$A_x = L_e^{-1/2} A^{(0)} + \frac{L_e^{3/2}}{r^2} \left( A^{(2)} - \frac{1}{2} \ln \frac{r}{L} \partial_t^2 A^{(0)} \right). \quad (3.4)$$

Then the conductivity can be expressed as [23]

$$\sigma = \frac{2A^{(2)}}{i\omega A^{(0)}} + \frac{i\omega}{2} - i\omega \log \frac{L_e}{L}, \quad (3.5)$$

where the factor of  $L_e^{-1/2}$  ensures that the gauge fields  $A_\mu^{(n)}$  have the correct dimensionality. We can obtain the conductivity by solving the motion equation (3.2) numerically for the general forms of function  $\mathfrak{F}(\psi) = \psi^2 + c_\gamma \psi^\gamma + c_4 \psi^4$ . Here we also focus our attention on the case for  $m^2 L^2 = -3$ .



TABLE II: The ratio  $\omega_g/T_c$  for different values of the Gauss-Bonnet constant  $\alpha$ , the model parameter  $c_\gamma$  and the Born-Infeld coupling parameter  $b$  with  $m^2 L^2 = -3$  and  $\gamma = 3$ .

$c_\gamma = 0$				$c_\gamma = 0.1$		
	b=0.0	b=0.1	b=0.2	b=0.0	b=0.1	b=0.2
$\alpha = 0.1$	8.5	9.0	10.0	9.3	9.7	11.6
$\alpha = 0.0$	7.7	8.1	8.7	8.4	8.7	9.7
$\alpha = -0.1$	7.3	7.6	7.9	7.8	8.1	9.0

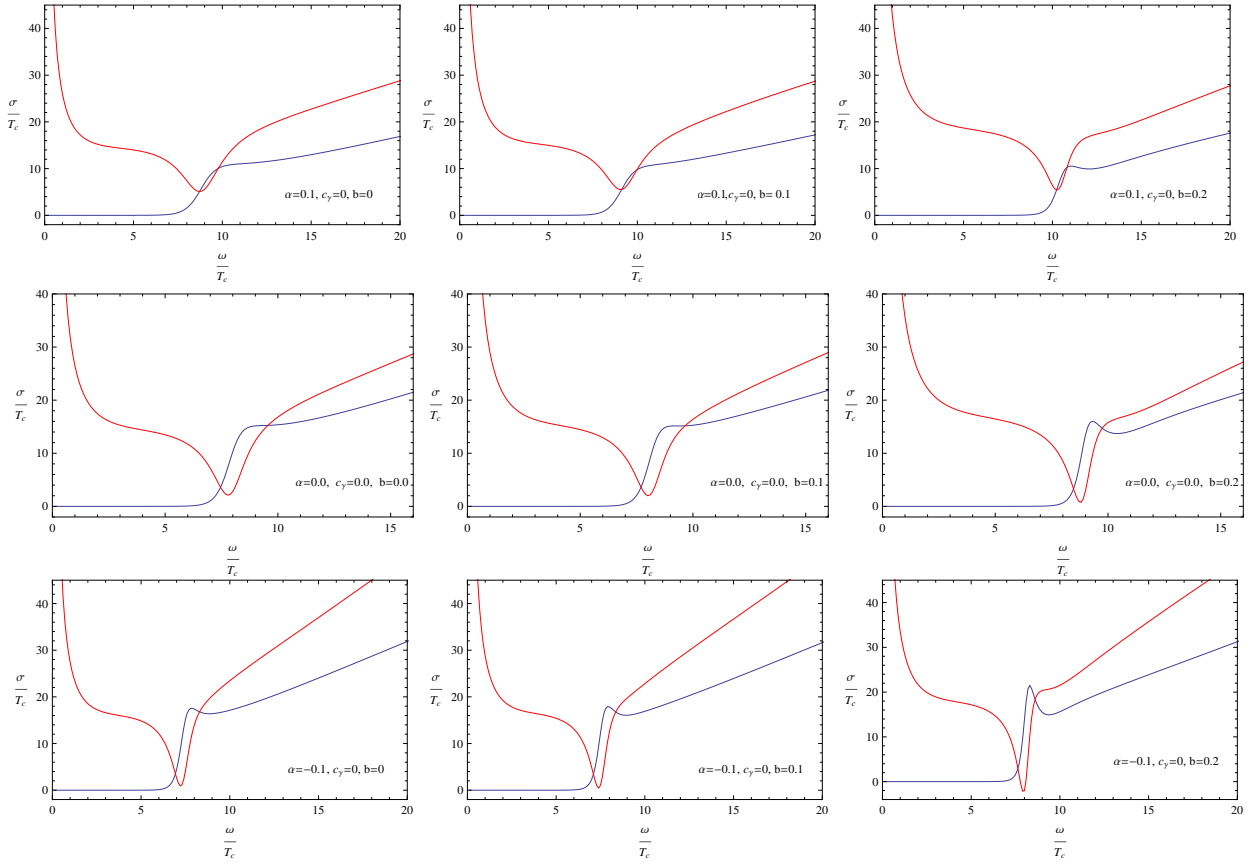


FIG. 2: (Color online) The conductivity of the superconductors as a function of  $\omega/T_c$  for different values of  $\alpha$  and  $b$  with  $c_4 = 0$ ,  $c_\gamma = 0$  and  $m^2 L^2 = -3$ . The blue (bottom) line represents the real part of the conductivity,  $Re(\sigma)$ , and red (top) line is the imaginary part of the conductivity,  $Im(\sigma)$ .

In Fig. 2 and table II we present the frequency dependent conductivity obtained by solving the motion equation of the Born-Infeld electrodynamics numerically for different values of  $\alpha$ ,  $b$  and  $c_\gamma$

with  $c_4 = 0$ ,  $\gamma = 3$  and  $m^2 L_{AdS}^2 = -3$  (we plot the conductivity at temperature  $T/T_c \simeq 0.3$ ). We find that the gap frequency  $\omega_g$  increases with the increase of the coupling parameter  $b$  for fixed  $\alpha$  and  $c_\gamma$ , it decreases as  $\alpha$  decreases for fixed  $b$  and  $c_\gamma$ , and it increases as  $c_\gamma$  increase for fixed  $\alpha$  and  $b$ . From Figs. 2 and table II, we find that the ratio of the gap frequency in conductivity  $\omega_g$  to the critical temperature  $T_c$  in the Gauss-Bonnet black hole with the Born-Infeld electrodynamics depends on the Gauss-Bonnet constant, the model parameters and the Born-Infeld coupling parameter.

#### IV. CONCLUSIONS

The behaviors of the holographic superconductors in the Gauss-Bonnet gravity have been investigated by introducing a complex charged scalar field coupling with an electric field obeyed to Born-Infeld electrodynamics in a planar black-hole background. We present a detail analysis of the condensation of the operator  $\langle O_+ \rangle$  by numerical method. For the interesting simple model  $\mathfrak{F}(\psi) = \psi^2 + c_4 \psi^4$ , we know that there is a phase transition from the second order to the first one as we alter the values of the Gauss-Bonnet constant  $\alpha$ , the model parameter  $c_4$  and the Born-Infeld coupling parameter  $b$ . For the transition point, the relation of  $\alpha$ ,  $c_4$  and the critical values  $b_c$  and  $T_c$  which can separate the first- and second-order behavior is: both  $b_c$  and  $T_c$  decrease as  $\alpha$  increases for fixed  $c_4$ , and  $b_c$  decreases but  $T_c$  increases as  $c_4$  increases for fixed  $\alpha$ . It is interesting to find that the Born-Infeld coupling parameter and model parameters have obvious different effects on the critical temperature for general model  $\mathfrak{F}(\psi) = \psi^2 + c_\gamma \psi^\gamma + c_4 \psi^4$ . If we fix the model parameters  $(c_\gamma, \gamma, c_4)$ , we note that the critical temperature becomes smaller as the Born-Infeld coupling parameter  $b$  increases for two types of phase transitions, i.e., the scalar hair can be formed harder for the larger  $b$ . However, the story is completely different if we fix the Born-Infeld coupling parameter  $b$ . For the cases of second phase transition, the formation of the scalar hair does not affect by model parameters  $(c_\gamma, \gamma, c_4)$ . But for the cases of first phase transition, the scalar hair can be formed easier for the larger model parameter  $(c_\gamma, c_4)$  or smaller  $\gamma$ . We finally find that the ratio of the gap frequency in conductivity  $\omega_g$  to the critical temperature  $T_c$  in the Gauss-Bonnet black hole with the Born-Infeld electrodynamics depends on the Gauss-Bonnet constant  $\alpha$ , model parameters  $(c_4, c_\gamma, \gamma)$ , and the coupling parameter  $b$ . Thus, the Gauss-Bonnet constant, model parameters and Born-Infeld coupling parameter provide richer physics in the phase transition and the condensation of the scalar hair.

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